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γ_5 transformations and supersymmetry

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Abstract. The algebra of supersymmetry transformations including a γ_5 transformation but not the full conformal group is studied and shown to imply strong restrictions on invariant actions. The connection to the R invariance of Fayet is made obvious.

Much work has recently been presented on the structure of supersymmetric Lagrangian models including gauge symmetries (Fayet 1974) and spontaneous symmetry breaking (Fayet 1975, O'Raifeartaigh 1975). It is apparent that the γ_5 transformations, originally part of the complete Wess and Zumino supersymmetry transformations (Wess and Zumino 1974a, Dondi and Sohnius 1974) play an important role. It is the purpose of this short note to include the γ_5 transformation in the supersymmetry subalgebra with the Poincaré group and an internal SU(n), and show the connection between this transformation and the R invariance introduced by Fayet (1974).

It is clear that the algebra of the supersymmetry generators, Poincaré group generators and SU(n) generators (Dondi 1975) can be enlarged to include a U(1) generator with

$$[T^5, Q^A] = -\tfrac{1}{2}Q^A \qquad A = 1, 2, \ldots, 2n \tag{1a}$$

$$[T^5, \bar{Q}_{\dot{A}}] = \tfrac{1}{2}\bar{Q}_{\dot{A}} \tag{1b}$$

(where the notation here and throughout is as in Dondi 1975) and all other commutators containing T^5 vanishing.

The action of the group element $R = \exp(\mathrm{i}T^5\alpha)$ on the manifolds

$$\phi(x, \theta, \bar{\theta}) = \exp(-\mathrm{i}xP + \mathrm{i}\theta Q + \mathrm{i}\bar{Q}\bar{\theta}) \tag{2a}$$

$$\phi_1(x, \theta\, \bar{\theta}) = \exp(-\mathrm{i}xP + \mathrm{i}\theta Q)\exp(\mathrm{i}\bar{Q}\bar{\theta}) \tag{2b}$$

$$\phi_2(x, \theta, \bar{\theta}) = \exp(-\mathrm{i}xP + \mathrm{i}\bar{Q}\bar{\theta})\exp(\mathrm{i}\theta Q) \tag{2c}$$

is simply

$$R\phi(x, \theta, \bar{\theta}) = \phi(x, \theta\,\mathrm{e}^{-\mathrm{i}\alpha/2}, \bar{\theta}\,\mathrm{e}^{\mathrm{i}\alpha/2})R \tag{3a}$$

$$R\phi_1(x, \theta, \bar{\theta}) = \phi_1(x, \theta\,\mathrm{e}^{-\mathrm{i}\alpha/2}, \bar{\theta}\,\mathrm{e}^{\mathrm{i}\alpha/2})R \tag{3b}$$

$$R\phi_2(x, \theta, \bar{\theta}) = \phi_2(x, \theta\,\mathrm{e}^{-\mathrm{i}\alpha/2}, \bar{\theta}\,\mathrm{e}^{\mathrm{i}\alpha/2})R. \tag{3c}$$

We extract the superfield transformation laws from the group multiplication in the usual way, so that we have

$$R\phi R^{-1} = \mathrm{e}^{\mathrm{i}r\alpha}\phi(x, \theta\,\mathrm{e}^{-\mathrm{i}\alpha/2}, \bar{\theta}\,\mathrm{e}^{\mathrm{i}\alpha/2}) \tag{4a}$$

$$R\phi_1 R^{-1} = e^{ir\alpha}\phi_1(x, \theta\, e^{-i\alpha/2}, \bar{\theta}\, e^{i\alpha/2}) \tag{4b}$$

$$R\phi_2 R^{-1} = e^{ir\alpha}\phi_2(x, \theta\, e^{-i\alpha/2}, \bar{\theta}\, e^{i\alpha/2}). \tag{4c}$$

Equations (4) show the connection between the R transformations defined by Fayet (1974), and the γ_5 transformations of Wess and Zumino (1974a). In fact, equations (1)–(4) are easily seen to be generalizations of the T^5 transformation given for a particular superfield with SU(2) symmetry by Dondi and Sohnius (1974).

As is well known, the general superfield can be reduced by constructing invariant constraints from the covariant derivatives. Thus, the constraint $\bar{D}_{\dot{A}}\phi(x, \theta, \bar{\theta}) = 0$, where $\bar{D}_{\dot{A}}$ is a covariant derivative, implies that $\phi_1(x, \theta, \bar{\theta})$ is independent of $\bar{\theta}$. Superfields of this type transform under R as

$$R\phi_1(x, \theta)R^{-1} = e^{ir\alpha}\phi_1(x, \theta\, e^{-i\alpha/2}). \tag{5}$$

This basic superfield can be used to construct Lagrangian models (Wess 1974, Firth and Jenkins 1975, Capper and Leibbrandt 1975, Dondi 1975). In the simplest case, when there is no internal SU(n) group, the action contains a kinetic term

$$A_{\text{kin}} \propto \int d^4x\, d^2\theta\, d^2\bar{\theta}\, \phi_1^+(x, \bar{\theta})\phi_2(x, \theta, \bar{\theta})$$

where

$$\phi_2(x, \theta, \bar{\theta}) = \exp(2i\theta\partial\bar{\theta})\phi_1(x, \theta)$$

is just the shifted superfield. This part of the action is easily seen to be R invariant for any value of r. The mass term in the action, given by

$$A_{\text{mass}} \propto \int d^4x\, d^2\theta\, d^2\bar{\theta}\, (\phi_1^2(x, \theta)\, \delta(\bar{\theta}) + \phi_1^{+2}(x, \bar{\theta})\, \delta(\theta))$$

transforms under R, as

$$\int d^4x\, d^2\theta\, d^2\bar{\theta}\, (\phi_1^2(x, \theta)\, \delta(\bar{\theta}) + \phi_1^{+2}(x, \bar{\theta})\, \delta(\theta))$$

$$\xrightarrow{R} \exp[i(2r-1)\alpha] \int d^4x\, d^2\theta\, d^2\bar{\theta}\, (\phi_1^2(x, \theta)\, \delta(\bar{\theta})) + \text{HC}$$

where HC stands for Hermitian conjugate, and therefore is only R invariant for $r = \frac{1}{2}$. This is so restrictive that an interaction term cannot be added without destroying the R invariance. Alternatively, an R invariant interaction, for example

$$A_{\text{int}} \propto \int d^4x\, d^2\theta\, d^2\bar{\theta}\, (\phi_1^3(x, \theta)\, \delta(\bar{\theta}) + \phi_1^{+3}(x, \bar{\theta})\, \delta(\theta))$$

with $r = \frac{1}{3}$, implies that the mass term breaks the invariance. Thus the simple massive scalar superfield Lagrangian with ϕ_1^3 interaction, which has such nice renormalization properties (Wess and Zumino 1974b) is not invariant under supersymmetry transformations when R is included.

The vector superfield of Wess and Zumino (1974a, c) is a superfield restricted only by the condition

$$V(x, \theta, \bar{\theta}) = V^+(x, \theta, \bar{\theta}) \tag{6}$$

and thus has an R transformation

$$V(x,\theta,\bar{\theta}) \xrightarrow{R} V(x,\theta\,\mathrm{e}^{-\mathrm{i}\alpha/2},\bar{\theta}\,\mathrm{e}^{\mathrm{i}\alpha/2}).$$

The last term in the expansion of the superfield as a function of the θ and $\bar{\theta}$ parameters is automatically R invariant, and this holds also for products of this type of superfield even when derivatives are included. Since it is just these terms which are used in the construction of a supersymmetric action, R invariance of the action is guaranteed.

When SU(2) as an internal symmetry is included, we have as the supersymmetric free action for the superfield constrained by $\bar{\mathrm{D}}_A\phi = 0$ (Capper and Leibbrandt 1975, Dondi 1975):

$$A = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathrm{d}^4\bar{\theta}\,[\phi_1^+\phi_2 - 2\phi_1(\Box + 2m^2)\phi_1\,\delta(\bar{\theta}) - 2\phi_1^+(\Box + 2m^2)\phi_1^+\,\delta(\theta)].$$

Here even in the case $m = 0$, the R invariance is only achieved for $r = 1$. This also ensures that the mass term is R invariant, but it is impossible to have an R invariant interaction.

Obviously, in the simple case of no internal SU(n) symmetry where the kinetic term of the scalar superfield does not force a choice of the r, it is possible, when one considers the interactions between more than one superfield, to construct non-trivial R invariant actions. Indeed, R invariance can be used to help restrict the possible couplings considerably by judicious choice of the r values (Fayet 1974, 1975, O'Raifeartaigh 1975).

When SU(2) is included, the situation is complicated by the fact that the kinetic term for superfields obeying $\bar{\mathrm{D}}_{\dot{A}}\phi = 0$ already restricts the choice of r. The possibility of using a superfield obeying equation (6) exists (Wess 1975), and like its simpler counterpart is less restricted by R invariance. Thus we have the situation of a symmetry which in the simple well explored models is too restrictive, but which seems to be very relevant when considering interactions between several superfields.

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